MATH 2850: PICARD'S THEOREM: EXISTENCE AND UNIQUENESS OF SOLUTIONS

PICARD'S THEOREM: Given the IVP: $y' = f(x, y)$, $y(x_0) = y_0$:

- If f is continuous on an 'open rectangle' containing (x_0, y_0) then there is at least one (possibly more than one!) solution to the IVP on some open interval containing x_0.
- If both f and f_y is continuous on an open rectangle containing (x_0, y_0), then the solution is unique.

EXAMPLE: Describe all points (x_0, y_0) for which the IVP $y' = 6x(y-4)^{2/3}$, $y(x_0) = y_0$ is guaranteed to have:

- at least one solution.

Ans: (x_0, y_0) can be any point in the plane.

- a unique solution.

Ans: (x_0, y_0) can be any point in the plane provided $y_0 \neq 4$.

EXAMPLE: Find two explicit solutions to $y' = 6x(y-4)^{2/3}$, $y(1) = 4$.

Ans: $y = 4$ and $y = (x^2 - 1)^3 + 4$

EXAMPLE: Apply Picard's EUT to the linear IVP: $y' + p(x)\,y = f(x)$, $y(x_0) = y_0$.

Assume p and f are continuous in an open interval containing x_0.

EXAMPLE: Apply Picard's EUT to the separable IVP: $h(y)\,y' = g(x)$, $y(x_0) = y_0$.

Assume h and g are continuous in open intervals containing x_0 and y_0, respectively, and $h(y_0) \neq 0$.

What additional hypotheses would be needed to ensure there were a **unique** solution to this IVP?

HOMEWORK: pg. 61: 1-13 odd, 14, 15*